Sample Weighting Methods and Estimation of Totals in the Consumer Expenditure Survey

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The widely used Principal Person method of weighting households in federal government surveys uses external post–Censal information on population to improve survey sample weights by a form of poststratification. While the Principal Person Methodology can be viewed as part of a procedure to adjust for nonresponse and undercoverage, it is not oriented for efficiently incorporating ancillary information or combining information from multiple surveys into survey estimates of subdomain totals In this article a generalized least squares adjustment algorithm is shown to incorporate ancillary information in a way that, in principle, reduces the design variance of estimated survey totals The flexibility of the method is exploited in an application to the Consumer Expenditure Survey that makes use of its "weighting control" and "composition" features

KEY WORDS Poststratification, Generalized regression estimators

1. INTRODUCTION

The prosaic subject of survey sample weighting is of some importance to the routine operations of large surveys such as the Current Population Survey (CPS) and the Consumer Expenditure Survey (CE) Estimates of number of persons by labor force status from the CPS, or number of consumer units (CU's) or economic families by tenure status from the CE, are of considerable public interest and are composed of sums of highly processed sample weights. The adjustments applied to these weights to account for nonresponse and to incorporate ancillary data are therefore of crucial importance to the quality of the estimates of totals produced.

For the purpose of making use of information about the target population that is ancillary to the survey, such as post–Censal population counts, it is generally the case that estimates can be produced directly for the purpose at hand without resorting to adjustment of the sample weights. There is considerable operational advantage in making weight adjustments that are beneficial in a mean squared error sense to arbitrary aggregations of the weights, however. In addition, it is often important to users of survey statistics that the estimates for various groups of households be consistent with one another, in the sense that they aggregate in an obvious way to higher-level totals. Because the Consumer Expenditure Survey is composed of two independent but parallel components, representing the same population but differing in survey instrument, another form of consistency is likely to be important to its users as well. The two components, a Diary survey and a quarterly Interview survey, produce comparable estimates of the number of consumer units for a variety of subdomains, since they have not only the same target population but also the same sampling frame. When these estimates differ for subdomains of particular and ongoing interest, such as tenure status or region of residence, it is desirable to be able to blend information from the two survey components to arrive at a "best" estimate. A direct composite estimate can usually be computed for these subdomains, but, again, performing a weighting adjustment that brings about consistency between the components in a way that usefully combines information would be preferable from the viewpoints of processing and user convenience.

This article examines some regression-based methods of survey weighting adjustment that satisfy these consistency criteria while producing estimates of totals with desirable statistical properties. The standard Principal Person weighting method is described first, and then the generalized least squares (GLS) method and several alternative approaches to weighting are discussed. The statistical underpinning of the GLS method in the generalized regression estimation literature is then reviewed. After a brief discussion of the problems of nonresponse and undercoverage in the context of the GLS method, new results are presented on using GLS to align totals compiled from identical variables collected in separate samples. A numerical example comparing variants of GLS with the Principal Person method is discussed, and then the same methods are applied to data from the early years of the Consumer Expenditure Survey. The findings of the empirical study are generally favorable to GLS as compared with the Principal Person method in the precision of the estimates of totals produced with the adjusted weights. In addition to

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making more effective use of ancillary population control total information, the composition or alignment feature incorporated into the GLS method here was found to be particularly beneficial. Frequent reference is made to the Consumer Expenditure Survey for setting a context and providing examples for the application of the techniques considered below. However, the principles outlined here transfer in one special case or another to a fairly wide class of surveys.

2. PRINCIPAL PERSON WEIGHTING

The Consumer Expenditure Survey has a multistage sample design, beginning with the selection of a sample of geographical areas from which housing unit addresses are drawn. The design sampling weights are determined according to the population of the sampled area and, within each area, to the number of housing unit addresses of each of seven subframes from which sample addresses are chosen, relative to the sample size allocated to the subframe. The weights are further stratified within each subframe. These design weights are adjusted using a "weighting control factor," accounting for, among other things, subsampling of unexpectedly clustered addresses—as, for example, in student housing built after the sample frame was constructed The people in each sample household are grouped into consumer units, the ultimate sample unit of the Consumer Expenditure Survey, and a further "noninterview" factor is applied to adjust for inability to obtain an interview from some consumer units in occupied households. Because the occupancy status of an address cannot be determined during the frame assembly process, vacant addresses that are by definition out of scope for the target population of the survey are deleted when encountered during the sampling process, and hence are not included in computing the consumer unit noninterview adjustment. This process of classifying the occupancy status of an address is inexact, and the noninterview adjustment yields Census-year estimates of total households that fall short of the number based on the Census of Population To correct for this coverage problem, a poststratification adjustment called Principal Person weighting is performed, using updated data on total persons in each of 48 age, race, and sex categories. These "control totals" make use of administrative records in births and deaths from local governments. The Principal Person adjustment begins with the assignment of the consumer unit weight to each person, followed by a second-stage ratio adjustment of these person weights to force the population in the 48 control categories estimated by the survey to equal the Census counts. A unique "principal person" is chosen to represent the CU, and this person's weight becomes the CU weight. The principal person is the female of a consumer unit head and spouse pair unless there is a single male head. If the principal person is male, his weight (and hence his CU's weight) is multiplied by a "principal person factor" to adjust for a historical tendency for males to be underrepresented compared with females The reason for designating the female as the principal person in this scheme is that the household weight is best determined as the weight of the "best covered" member when coverage of people within a household is incomplete. The Principal Person methodology is described in Hanson (1978, chap. V). Alexander (1986) provides an excellent review.

The performance of Principal Person weighting might best be assessed by examining the mean squared error (MSE) of the statistics computed with the adjusted weights. For totals, minimizing the coverage errors remaining after the noninterview adjustment is tantamount to minimizing the bias component of the MSE. To the extent that the Principal Person methodology achieves its primary goal, Principal Person totals should perform well on this score. The variance component of the MSE of Principal Person totals also benefits from the ratio adjustments incorporating population control totals. Additional variance control is achieved by collapsing small poststratification cells according to ad hoc cell-adjacency criteria. While for large surveys it is reasonable to suppose that the bias component introduced by undercoverage dominates, this will be less true for smaller surveys. In any event, experience with the early years of the Consumer Expenditure Survey has raised concern about the ability of current weighting methodology alone to deal effectively with even the bias component, let alone the variance component, of the MSE. Certain subdomains, such as total CU's, total homeowner CU's, and total single-person CU's, have often differed noticeably and significantly between the Diary and Interview survey components on a quarter-by-quarter basis. Even if the traditional weighting adjustment procedures are the best available in dealing with the coverage problem, it would be useful to augment them with methods designed to align comparable survey totals while controlling variance through the efficient use of ancillary data such as post–Censal population counts.

3. WEIGHTING CONTROL PROCEDURES

3.1 Luery–Roman Alternative to Principal Person Weighting Adjustment

In this section, to illustrate a principle upon which possibly superior estimators of population and CU totals can be constructed, a weighting adjustment procedure is described that was introduced by Luery (1980)

To proceed, some notation is required Let

$\Omega = n \times 1$ vector of design sample weights for a sample of n sample units, representing the inverse of the design probabilities of selection, π, from a population of $\mathfrak{N}$ units;

$X = K \times n$ matrix of control characteristics of each sample unit whose aggregate population values are known with certainty, such as number of persons in K cells defined by age, race, and sex in each consumer unit;

$N_X = K \times 1$ vector of control counts of aggregate population values of characteristics X that are taken to be known with certainty, such as number of persons in the population in cells defined by age, race, and sex;

$W = n \times 1$ vector of adjusted weights; and

$\Lambda = n \times n$ weighting matrix.

Both Luery (1980) and Roman (1982) assumed either $\Lambda = \mathrm{diag}(\Omega)$ or $\Lambda = (\mathrm{diag}(\eta))^{-1}\mathrm{diag}(\Omega)$, where each element of the $n \times 1$ vector η is the total of the columns of X for each sample unit. In the current Consumer Expenditure Survey context, X and N_X have $K = 48$ rows corresponding to the age/race/sex classifications for which there are control counts from administratively updated Census data. In practice, Ω includes adjustments for unit nonresponse, and is sometimes referred to as the "unbiased weight" by survey practitioners because differential nonresponse bias has presumptively been removed from the weights by the adjustment.

It is desired to change the unbiased weights Ω as little as possible so that the sample weighted sums of characteristics are the same as the control counts. Consider

$$\min_W(\Omega - W)'\Lambda^{-1}(\Omega - W), \qquad (3.1)$$

subject to $X'W = N_X$. The solution of this problem yields

$$\hat{W} = \Omega + \Lambda X(X'\Lambda X)^{-1}(N_X - X'\Omega), \qquad (3.2)$$

and we have assigned sample unit weights accounting for sample unit characteristics on which there is population control information that exactly aggregate to the control totals N_X.

Because of the form of the objective function for the adjustment of the sample weights, in this and subsequent sections the Luery–Roman and related methods are referred to as GLS methods. Other objective functions for adjusting the weights have been studied, including the minimum discriminant information criterion resulting in the well-known iterative proportional fitting or raking algorithm of Deming and Stephan (1940), the maximum likelihood criterion corresponding to multinomial sampling, and the minimum chi-squared criterion of Stephan (1942) and Smith (1947). This last approach is essentially the same as the Luery–Roman except that Λ depends on W instead of Ω. The minimum chi-squared method was used in "aging" CPS data at the Social Security Administration in the early 1970s using a methodology developed by Pugh, Tyler, and George (1976). The raking method was the subject of an extensive empirical analysis by Scheuren, Oh, Vogel, and Yuskavage (1981) on adjusting person weights using CPS data. GLS was the subject of similarly intensive testing for adjusting consumer unit weights by Zieschang (1985, 1986a,b). Bethlehem and Keller (1983, 1987) and Lemaître and Dufour (1988) discuss a prediction-oriented least squares method closely related to GLS. A review and an evaluation of several of these methods is contained in Alexander (1988). Until recently, GLS had probably received the least attention, though it will be shown below that it is grounded in the recent literature on generalized regression estimation of finite population statistics

Sample weighting has been the focus in recent years of a steady stream of algorithmic studies. Fagan and Greenberg (1984, 1985) provided a review of the algorithmic literature as well as contributing to resolving some computational issues with the leading algorithms Of the adjustment algorithms named, GLS is probably the most computationally straightforward because it is not iterative, a potential advantage when processing large surveys in the face of rigid publication schedules.

In the Consumer Expenditure Survey context, the adjusted weights W include a term that is a function of the person composition of each CU according to the age/race/sex control information on numbers of persons that is available for the target population. Note that if the K categories are for attributes unique to each sample unit, such as the region in which the unit is located, and the GLS weighting matrix is $\Lambda = \mathrm{diag}(\Omega)$, this problem resolves to the simple ratio adjustment of weights of CU's in those attribute cells so that their sums equal the control totals for those cells. In the CE context, since no controls exist for CU totals this simple ratio adjustment cannot be performed The Luery–Roman procedure represents a generalization of simple ratio adjustment to accommodate sets of attributes (composition by types of persons) that do not classify sample units (CU's) into mutually exclusive groups

3.2 Function of Weighting Control Procedures in the Estimation of Population Totals: A Model-Based Approach

If Y is an $n \times L$ matrix of characteristics of each sample unit corresponding to another set of attributes for which no control information exists and if estimates for the population are desired, the estimate that results from GLS weighting control is $\tilde{N}_Y = Y'\hat{W}$. The components of $\tilde{N}_Y$ could include CU totals as in the preceding paragraph. The properties of this estimator are of primary interest Some analytical results on the properties of GLS weighting controlled estimators can be developed from the literature on generalized regression estimation.

The above estimator $\tilde{N}_Y = Y'\hat{W}$ of the $L \times 1$ population vector N_Y can be written as

$$\begin{aligned}\tilde{N}_Y &= Y'[\Omega + \Lambda X(X'\Lambda X)^{-1}(N_X - X'\Omega)] \\ &= \hat{N}_Y + \hat{\beta}'(N_X - \hat{N}_X),\end{aligned}$$

where

$$\hat{N}_Y = Y'\Omega, \qquad \hat{N}_X = X'\Omega, \qquad \hat{\beta} = (X'\Lambda X)^{-1}X'\Lambda Y, \qquad (3.3)$$

and $\hat{\beta}$ is of dimension $L \times K$. As recognized by Bethlehem and Keller (1983) and Luery (1980, 1986), $\tilde{N}_Y$ is a regression estimator. In fact, it is a version of the generalized regression estimator of Cassel, Sarndal, and Wretman (1976). The generalized regression estimator is in turn a member of the QR class of estimators identified by Wright (1983), which have the form

$$\tilde{N}_{QR} = \hat{\beta}'N_X + \varepsilon'_{YX}r,$$

where

$$\varepsilon_{YX} = Y - X\hat{\beta}, \qquad \hat{\beta} = (X'\mathrm{diag}(q)X)^{-1}X'\mathrm{diag}(q)Y,$$

and q and r are known $n \times 1$ vectors. Wright shows that $\tilde{N}_{QR}$ generalizes a wide variety of proposed regression-type and ratio estimators, including those of Royall (1970), Cassel et al. (1976, 1977), Brewer (1979), Sarndal (1980a,b), Isaki and Fuller (1982), and others. The generalized regression class is defined for $r = \pi^{-1}$ and arbitrary q.

The $L \times K$ matrix $\hat{\beta}$ can be seen as an estimator of the parameter matrix β for the following linear model ξ relating Y to X:

$$Y = X\beta + \varepsilon, \tag{3.4}$$

where

$$E_{\xi}(Y) = X\beta, \quad \operatorname{cov}_{\xi}(\varepsilon_k) = \Sigma_k, \quad W\Sigma_k^{-1} = W\Sigma_l^{-1} = \Lambda,$$

for $k, l = 1, 2, \ldots, L$, where ε_k is the kth column of the $n \times L$ matrix ε, W is a diagonal weighting matrix, Σ is a diagonal covariance matrix, and the subscript ξ refers to moments taken with respect to the model. In the model ξ, Y is considered as a realization of n row vectors from a finite population of $\mathfrak{N}$ random row vectors of dimension $1 \times L$, independent across the population labels 1, 2, $\ldots$, $\mathfrak{N}$, denoted here as $\mathfrak{U}$. The properties of the generalized regression estimator of linear statistics for finite populations under a linear superpopulation model such as ξ for various sample designs were extensively studied by Cassel et al. (1976, 1977), Sarndal (1980a,b), Wright (1983), Robinson and Sarndal (1984), and others. The special case of the model generated by the GLS weighting algorithm of this article is very similar in form to the homoscedastic "Horvitz–Thompson" model of Särndal (1980b), so that $\Lambda = W$ and $W = (\operatorname{diag}(\pi))^{-1}$, where π is an $n \times 1$ vector of sample inclusion probabilities. The weighted least squares sample weighting algorithm of Luery (1980) is consistent with this model. Lemaître and Dufour (1988) note that the Bethlehem and Keller (1983, 1987) variant of least squares weighting obtains when the X matrix contains a column of ones, or, equivalently, when the model (3.4) has a constant term. The exact approach used in this paper and outlined below was developed in Zieschang (1985, 1986a,b) by viewing Λ as the randomization distribution covariance matrix of the sample weights prior to adjustment, essentially the same covariance matrix used by Bethlehem and Keller (1983). Särndal (1980b) established that the generalized regression estimator under the model ξ is design consistent in the sense of Brewer (1979) under a design where the inclusion probabilities vector π is proportional to the square root of the diagonal of Σ. He established that, though $\tilde{N}_Y$ is generally design biased, it is asymptotically design unbiased and efficient when the design and the model are matched in this way.

Note that essentially all of the literature on generalized regression estimation deals with the case $L = 1$, so that the matrix Y is composed of a single column vector. An inherent restriction of regression estimators for totals N_Y that are developed from GLS weighting algorithms is that $W\Sigma_l^{-1} = W\Sigma_k^{-1}$ for $k \neq l$, that is, Σ is identical for every possible model, when $L \geq 2$. In fact, most of the proposed regression estimators for β satisfy this restriction anyway, though an interesting exception can be found in Fuller (1975).

Fuller and Isaki (1981) and Isaki and Fuller (1982) developed a variant of the QR estimator for totals and means, having the form

$$\tilde{N}_Y^{\mathrm{IF}} = \hat{N}_Y + \hat{\beta}^{\mathrm{IF}\prime}(N_X - \hat{N}_X)$$

under the model

$$Y = [\pi \quad X]\beta + \varepsilon, \tag{3.5}$$

in which the first column of the regressor matrix is the vector of inclusion probabilities π. The estimator for $\beta = (\beta^{\pi\prime}, \beta^{\mathrm{IF}\prime})'$ is of the form (3.3) with X replaced by the regressor matrix in model (3.5), and with $\Lambda = (\operatorname{diag}(\pi))^{-1} = (\operatorname{diag}(\Omega))^2$. $\tilde{N}_Y^{\mathrm{IF}}$ is design consistent and asymptotically design efficient for a broad class of designs with the exception of, for example, stratified samples in which the number of strata increases in direct proportion to the sample size.

The Isaki–Fuller estimator can also be motivated by an appeal to a "minimum squared sample weight adjustment" criterion. Consider the problem (3.1) with the constraints

$$\begin{bmatrix} \pi' \\ X' \end{bmatrix} W = \begin{bmatrix} n \\ N_X \end{bmatrix}. \tag{3.6}$$

The solution $\tilde{W}$ of this minimization can be written as

$$\tilde{W} = \Omega + \Lambda Z(Z'\Lambda Z)^{-1} \begin{bmatrix} n - \pi'\Omega \\ N_X - X'\Omega \end{bmatrix}, \tag{3.7}$$

where

$$Z = [\pi \quad X]$$

and $n - \pi'\Omega = 0$ by definition of Ω.

Wright (1983, ths. 1 and 2) established that, in the notation of this paper, the Isaki–Fuller estimator for N_Y is equivalent to the generalized regression estimator

$$\tilde{N}_Y^{\mathrm{IF}} = \hat{N}_Y + \tilde{\beta}^{\mathrm{IF}\prime}(N_X - \hat{N}_X),$$

where $\tilde{\beta}^{\mathrm{IF}}$ is an estimator of the parameters of the model

$$Y = X\beta + \varepsilon \tag{3.8}$$

of the form (3.3), with the same assumed matrix Λ as model (3.5). Wright (1983) established that including functions of the selection probabilities π in the model regressor matrix can be used to ensure either asymptotic consistency or asymptotic design unbiasedness (ADU) for any QR estimator, including the Horvitz–Thompson and Isaki–Fuller types considered above. Wright further established that all QR estimators have the Cassel–Särndal–Wretman generalized regression form under the ADU property, so that under ADU, variants of the QR such as the Horvitz–Thompson, Isaki–Fuller, and Royall estimators come down to variations on the form of Λ. All of these results are predicated on enough prior knowledge of the population sampled, in particular regarding the response characteristics of the individual units, that the design inclusion probabilities π are the realized inclusion probabilities of units in the sample. Since this level of prior knowledge is rarely achieved in practice, a discussion of some sources of failure in the realization of the design is warranted.

3.3 Nonresponse and Undercoverage

It should be noted that most, though not all, studies of GLS weighting and the generalized regression estimator have concentrated on variance reduction and assumed that the design inclusion probabilities π are positive and accurate for every member of the population of interest, ignoring the systematic shortfalls of person counts relative to control totals that are endemic in survey operations. Most obviously, this can be a result of nonresponse. In the Consumer Expenditure Survey, nonresponse adjustments are made to compensate for this by ratio adjusting the weights in cells defined by geography, CU size, race of head, and tenure status, to cover nonrespondents, who are classified through information collected from neighbors. Undercoverage of the sample weights, thus corrected, can still occur if some of the nonrespondent in-scope sample units are misclassified as out of scope, as when an occupied address is miscoded as vacant This misclassification is similar to an error in the frame for the target population Other sources of undercoverage can arise from the process by which frames are updated. The central frame for the CE is the address list from the Decennial Census. The frame can therefore be as old as 10 years or more at the time the CE sample is collected. To compensate for this, addresses are updated by recanvassing "area segments"—geographical districts in which there are an unacceptably high proportion of sample addresses that cannot be located or have been demolished—and through examination of administrative (local government) records on permits issued for new construction. It is nevertheless evident from comparing post–Censal data on total urban addresses with address counts estimated using the nonresponse-adjusted survey weights that significant coverage errors remain

Luery (1986) discussed the coverage model implicit in the Horvitz–Thompson variant of GLS weighting adjustment algorithms when undercoverage exists. Alexander and Roebuck (1986) discussed the coverage issue for this and several non-GLS adjustment algorithms as well. Studies of this type are relevant, because it is rarely the case that completely accurate adjustments for frame coverage can be made prior to the incorporation of the ancillary control information in N_X at the final stage of weighting Following Wright (1983), the result that all QR estimators are generalized regression estimators under the desirable ADU property implicitly requires, at a minimum, the accuracy of the set of selection probabilities π. This will likely not be the case in actual survey estimation contexts because of the existence of differential survey nonresponse and frame errors as discussed above (wherein the number of successful sample hits, say n_s, is less than the designed sample size n), and the fact that nonresponse adjustments are either absent or inadequate. Under these circumstances, the design inclusion probabilities π are not the realized inclusion probabilities, say $\pi^* \leq \pi$, that should be the coefficients of the ADU constraint $\pi'W = n_s$ in system (3.6). This view of nonresponse and undercoverage echoes Oh and Scheuren (1983, p. 144): ". . the missing data problem is handled as if it were a sample design problem where some of the information about the selection probabilities is unknown." The widely used "weighting cell" methods of nonresponse adjustment attempt to estimate π^* directly, by expanding the weights of respondents in each of a set of, say, R cells with a ratio adjustment to cover both respondents and nonrespondents. The cells are formed by grouping units (CU's) by measured characteristics (typically demographic) believed to be related to their response behavior to the survey instrument. The empirical study below takes this approach by treating (weighting cell) nonresponse-adjusted weights as data for GLS weighting adjustment to control totals. Sarndal and Hui (1981) point out that overlaying the response mechanism on the sample design implies adjustment to the covariance matrix of the generalized regression estimator according to inverse probabilities of response; that is, Λ should be a function of π^* instead of π Under the Sarndal–Hui approach, the weights used as data for adjustment to control totals would be first adjusted by the estimated response probabilities according to a model prior to the GLS weighting adjustment. Since the Principal Person adjustments could be viewed as a part of the existing nonresponse adjustment process, this would provide justification for overlaying GLS weight adjustment on the Principal Person weights.

An alternative and novel method for nonresponse adjustment within the GLS framework—inspired by the weighting cell approach—would include an adjustment for differential nonresponse for all cells in a single computation Let there be R nonresponse cells. Let the design sample size be adjusted for frame errors (which will be at least as large as the number of respondents) in each nonresponse cell by n_r. In the GLS weighting problem (3.1), include a control constraint for each cell r of the form $(\Omega_r^{-1})'W = n_r$ $(r = 1, \ldots, R)$, where $\Omega_{r,i} = \Omega_i$ if unit i is in cell r, $\Omega_{r,i} = 0$ otherwise, and $\sum_{r=1}^{R} n_r = n$. This is merely an elaboration of the Isaki–Fuller ADU constraint $\pi'W = (\Omega^{-1})'W = n$. This adjustment would still be distinct from a subsequent GLS weighting adjustment incorporating control totals, because population coverage errors are likely to remain, even after adjustment for differential nonresponse

In the empirical section below, for all but one alternative variant a GLS adjustment is performed on the sample weights after the production weighting cell nonresponse adjustments—but before the Principal Person adjustments—used in the Consumer Expenditure Survey are made. As pointed out by Oh and Scheuren (1983) and Alexander and Roebuck (1986), the coverage properties of the final weighting adjustments depend on the accuracy of the nonresponse/coverage model inherent in the particular adjustment method used.

3.4 Including Composite Constraints

An interesting, new, and useful generalization of the constraints system on which the foregoing discussion of GLS weighting is based arises in the context of equating estimates of comparable totals between two surveys. An example of this context is the aligning of the estimates of consumer unit counts for a variety of subdomains between the Diary and Interview components of the Consumer Expenditure Survey Letting subscripts 1 and 2 refer to the two surveys, respectively, consider the sample weight adjustment problem (3.1) under the following definition of the principal variables

$$\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix},$$

$$X = \begin{bmatrix} X_1^c \\ -X_2^c \end{bmatrix}, \qquad N_X = 0. \tag{3.9}$$

In this version of the GLS problem, each survey's sample weights are adjusted to equate or make composites of the estimates of totals between surveys for the subdomains c. The generalized regression estimator for differences between the survey estimates of totals of units with characteristics Y, $\Delta^Y = \tilde{N}_1^Y - \tilde{N}_2^Y = [Y_1' - Y_2']' \tilde{W}$, where $\tilde{W}$ is computed from Equation (3.2) with substitutions (3.9), is

$$\Delta^Y = [\tilde{N}_1 - \tilde{N}_2] = [\hat{N}_1^c - \hat{N}_2^c] + \hat{\beta}^{Y\prime}(\hat{N}_2^c - \hat{N}_1^c).$$

In the Appendix it is shown that $\hat{\beta}^Y$ is the GLS regression estimator

$$\hat{\beta}^Y = \left([X_1^{c\prime} \quad X_1^{c\prime}] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ -\Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \right)^{-1}$$

$$\times [X_1^{c\prime} \quad X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ \Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

corresponding to the model ξ^c for the additional characteristics Y,

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \beta^Y + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix},$$

where

$$E_\xi \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \operatorname{cov}_\xi \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}' & \Sigma_{22} \end{bmatrix} = \Sigma,$$

and where $i, j = 1, 2, \ldots, N$, and $\Lambda = W\Sigma^{-1}$ for diagonal W, Σ. The diagonality of Σ presumes first that the surveys are independent draws from the same population or draws from different populations, and second that there is no model dependency between elements in the samples In the first case, the off-diagonal blocks of Σ satisfy $\Sigma_{21} = \Sigma_{12}' = 0$, and in the second, the blocks on the main diagonal are diagonal. The second assumption could be deemed unreasonable if two or more observations are taken from individual sample units, as in a panel design, and unit behavior is correlated over time. In fact, such a case is encountered in the empirical study below The diagonality of W requires first that there must not be any ultimate sample units in common between the surveys, and second that units must be independently selected within each survey. Under a panel design, W becomes nondiagonal, because the joint inclusion probability of two observations on a sample unit is the same as the unit's inclusion probability.

In Zieschang (1985) it is shown how the estimator for N^c resulting from the composite GLS weighting problem has the form of an empirical James–Stein multivariate composite estimator of the totals N^c. This is evident from the form of the generalized regression estimator for composite totals N^c for, say, survey 1

$$\tilde{N}_1^c = \hat{N}_1^c + \hat{\beta}_1^{cc\prime}(\hat{N}_2^c - \hat{N}_1^c) = (I - \hat{\beta}_1^{cc\prime})\hat{N}_1^c + \hat{\beta}_1^{cc\prime}\hat{N}_2^c,$$

where

$$\tilde{N}_i^c = X_i^{c\prime}\tilde{W}_i, \qquad \hat{N}_i^c = X_i^{c\prime}\Omega_i,$$

for $i = 1, 2$, and, by satisfaction of the constraints of the weighting adjustment problem, $\tilde{N}_1^c = \tilde{N}_2^c$ $\hat{\beta}_1^{cc}$ can be written

$$\hat{\beta}_1^{cc} = \left([X_1^{c\prime} \quad X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & 0 \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \right)^{-1} X_1^{c\prime}\Lambda_{11}X_1^c$$

$$= [X_1^{c\prime}\Lambda_{11}X_1^c + X_2^{c\prime}\Lambda_{22}X_2^c]^{-1} X_1^{c\prime}\Lambda_{11}X_1^c.$$

Hence

$$\hat{\beta}_1^{cc} = [\Sigma_{\hat{N}_1^c} + \Sigma_{\hat{N}_2^c}]^{-1}\Sigma_{\hat{N}_1^c},$$

where $\Sigma_{\hat{N}_i^c} = X_i^{c\prime}\Lambda_{ii}X_i^c$ is the covariance matrix of the total estimator $\hat{N}_i^c$ conditional on sample i. $\hat{\beta}_1^{cc\prime}$ is therefore the value of the weighting matrix A that minimizes the sample conditional variance of the composite $(I - A)\hat{N}_1^c + A\hat{N}_2^c$ in the sense of minimizing the quadratic form

$$\gamma'[I - A' \quad A'] \begin{bmatrix} \Sigma_{\hat{N}_1^c} & 0 \\ 0 & \Sigma_{\hat{N}_2^c} \end{bmatrix} \begin{bmatrix} I - A \\ A \end{bmatrix} \gamma$$

for arbitrary γ.

The case considered in the empirical section below combines the control constraints of Section 3 1 with the composite constraints of this section The principal variables of the weighting adjustment problem (3.1) are redefined as

$$\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix},$$

$$X = \begin{bmatrix} X_1^0 & 0 & X_1^c \\ 0 & X_2^0 & -X_2^c \end{bmatrix}, \qquad N_X = \begin{bmatrix} N_{X^0} \\ N_{X^0} \\ 0 \end{bmatrix}, \tag{3.10}$$

where Λ_{11} and Λ_{22} are assumed (block) diagonal, $\Lambda_{12} = \Lambda_{21}' = 0$, the superscript c indicates domains whose counts are to be equated by the weighting algorithm, and the superscript 0 now distinguishes characteristics X whose population aggregates N are known with certainty.

3.5 Negative Weights

There is nothing in formula (3.7) for the adjusted weights $\tilde{W}$ precluding them from being 0 or negative. This is not a problem regarding the statistical properties of the GLS weights discussed above, but it may engender some discomfort when viewing the weights as representing inverse probabilities of inclusion in the sample For users of production micro data files, further processing designed to limit the range of adjustment of GLS may be desirable. There are three methods of dealing with this problem. The first is to expediently recode the adjusted weight when it falls outside a tolerance interval containing the unadjusted sample weight. The second is to bound the adjustments to the weights within preset tolerance intervals explicitly using quadratic programming methods. The third method, reported in Bankier (1990), iteratively halves the elements of the diagonal of Λ corresponding to sample units (here, CU's) for which the GLS-adjusted weight is either too large or too small according to a set of prior limits. The effect of this is to penalize adjustment of the weights for these units in meeting the control counts. In his application to Canadian census data, Bankier noted that as many as 10 iterations were required to meet the limits he specified. In the interest of reducing computational burden, only the "expedient" method was examined in the empirical study below. Zieschang (1985) checked the method as implemented here against results where no recoding was done, and found only a minor, essentially negligible, impact on the estimates of totals and their variances. On this evidence, the recoding method works satisfactorily as long as the problem is specified and the limits to adjustment are set so that recoding is infrequent. Research on computationally efficient implementations of the programming or Bankier approaches to bounding weight adjustments is clearly desirable

4. AN ARTIFICIAL EXAMPLE DEMONSTRATING GLS

To show how GLS weighting adjustments work, an artificial example may be helpful. Consider the data given in Table 1, in which there is undercoverage of females in a small sample of households containing males and females. We might take the context of this example as a survey's coverage of occupied addresses, where we would encounter the problem of correctly classifying an occupied address as occupied when it has been impossible to obtain an interview. Practically, the example focuses on a further adjustment to sample weights after nonresponse or noninterview adjustments have been applied. Three techniques are displayed in Table 1: Principal Person, GLS, and GLS Over Principal Person.

In the Principal Person example, the CU base weight is assigned to each person in the CU, persons are divided into male/female cells, and the totals in these cells are ratio adjusted to the control totals of 300 for each person type Person weights within each CU are then assigned as the CU base weight multiplied by the corresponding person-type ratio. The weight of the CU is then assigned as the person weight for the person type with the best coverage, here, the male for male-only and male/female CU's, and the female for female-only CU's. A second column under Principal Person gives the weights under a further adjustment to female person weights in female-only CU's that equates the weighted number of "reference person" and "spouse" persons in male/female CU's. Since females are undercovered in this example, their person weights are inflated so that the number of married females is equal to the number of married males. The Principal Person adjustment might be considered a coverage adjustment designed to account for CUs totally missed because none of their members were available for interview and their address mistakenly classified as vacant and out of scope. It can be seen in Table 1 that either with or without the "marriage adjustment," the Principal Person weights do not meet the control totals for persons.

In the GLS example, formula (3.2) was applied to the base weights Ω to obtain a set of CU weights adding to the person control totals, resulting in an estimate of total CU's (for which there is no control total) roughly similar to the Principal Person estimate without marriage adjustment. The GLS Person Weighted column of the table also demonstrates the effect of setting up the GLS problem as adjusting *person* weights to meet the control totals in such a way that all person weights within each CU are the same (though they are permitted to be different for the same

Table 1 Alternative GLS Weighting Adjustment Methods on Artificial Data

CU	*Unit weight* Ω	*Male*	*Female*	*Principal Person (PP)*[a]	*GLS*	*GLS Person Weighted*	*GLS Person Weighted Over PP*[b]
1	50	1	1	53 571/53 571	67 301	65 407	58.796/57 665
2	50	1	0	53 571/53 571	46 655	47 965	51 097/51 632
3	30	1	2	32 143/32 143	52 768	42 733	36 818/35 805
4	40	0	1	60.000/84 000[c]	56 517	66 279	74 475/75 877
5	50	2	0	53 571/53 571	43 310	47 965	51.097/51 632
6	50	0	1	75 000/105 000[c]	70.646	82 849	93 094/94 847
7	50	1	0	53 571/53 571	46 655	47 965	51 097/51 632
CU totals	320			381/435	384	401	416/419
Persons	280 M, 200 F			300 M, 253 F/300 M, 307 F	300 M, 300 F	300 M, 300 F	300 M, 300 F

[a] The principal person is the male in CU's with a reference person (household head) and a spouse, and is the reference person otherwise, because in this example (contrary to actual *implementations of the Principal Person method) males are the best-covered person types Under the Principal Person method, the CU is assigned the weight of the principal person*

[b] Computed over Principal Person weights without marriage adjustment/with marriage adjustment

[c] Multiplied by a "marriage adjustment" factor Person weights are first computed by assigning the consumer unit weight to each person in each CU, and ratio adjusting those weights to meet person control totals for males and females Then the weighted number of married males is divided by the weighted number of married females, which determines the factor in such a way that if it were applied to the weights of married females, their number would equal the estimated number of married males This within-CU female undercoverage factor is then assumed to hold for all CU's, but only affects the CU weight for CU's with female principal persons It should be emphasized that females have been taken as the undercovered person type for the sake of this example only In the survey data from real populations to which this technique is applied, males are undercovered, and the marriage adjustments are applied to the weights of households with single male heads

person type between CU's). On this variant, mentioned in Section 2, see Luery (1980, 1986) and Alexander (1987). This results in an estimate of total CU's about halfway between the without- and with-marriage adjustment estimates from the Principal Person column. The last column demonstrates the application of the GLS technique when the base weights Ω are taken as the Principal Person weights, resulting in the total CU estimates below, but still closer to the Principal Person estimate with marriage adjustment This illustrates the case mentioned previously in Section 3.3 on Nonresponse and Undercoverage, where Principal Person adjustments are taken as self-consciously adjusting for a particular response process in the data before GLS is used to adjust the weights to the control totals.

This is, of course, only an example. More informative results on comparing the GLS and Principal Person methods taking the sampling/response process into account might be obtained from simulations under specific hypotheses about the response process.

5. APPLYING GLS WEIGHTING TO THE CONSUMER EXPENDITURE SURVEY

5.1 Specification

The weighting adjustment procedure characterized by problem (3.1) with definition of variables (3.10) was applied to quarterly Consumer Expenditure Survey data for the period 1980IV–1983IV. To evaluate the performance of the GLS procedure in the context of the Consumer Expenditure Survey, the time interval for weighting was set at a quarter. Current Principal Person procedures are implemented on a monthly basis. However, this results in weighting batches of consumer units of between 300 and 400 CU diary-weeks for the Diary survey, or about the same number of CU interviews for rotation groups in the Interview survey, which has a rotating panel design The number of CU's drops to 150–200 for the sample replicates (used for variance calculation) that are weighted in parallel with the full sample. Batches of this size tend to have patchy coverage of the 48 CU member age/race/sex characteristics that are to be controlled to the Census counts Current procedure deals with this by a large amount of ad hoc collapsing of control cells into one another, sacrificing control detail. To avoid this at the outset, the data were aggregated into quarterly batches for weighting purposes. However, to ensure that the Diary months and Interview rotation group/months are correctly scaled relative to one another, the GLS constraints were set up to control each month's or rotation/month's CU member weights to sum to the monthly total population. Despite quarterly weighting, the number of control cells for the Diary survey was further reduced by aggregating the 12 available age categories into six owing to the presence of empty cells in either the full sample or the replicates The number of Interview cells remained at 48. Table 2 provides a description of the age/race/sex control cells used for the two surveys. The age/race/sex control counts equaled the average quarterly population for each cell. The monthly control counts were set at a third of the corresponding monthly totals for the Diary and a twelfth of the monthly totals for the Interview so that the sum of controls across the three months in the first case and across the three months and four rotation groups in the second case would sum to average total population for the quarter.

The composite subdomains chosen for the columns of the X^{i} matrices of Section 3.4 include region of residence, sampling frame from which the CU was drawn, tenure status of the CU, and four family types. These groupings are described in Table 3. Region, tenure, and family type were chosen as composite subdomains because they correspond to an aggregation of the classification system on which CE statistics are published The frame classifications were included because they are a part of the stratification scheme of the sample design, and form CU strata that are known to be the same size by definition. More important, differences in the frame totals between surveys could reasonably be expected to be correlated with differences in totals on other subdomains

The weighting constraints used for this empirical study correspond in general form to those of problem (3.1) with variable definitions (3 10), where sample 1 is the Diary sample for a given quarter and sample 2 is the Interview. The matrix X_1^{0} therefore has 26 columns of which 24 correspond to the Diary classification of persons for quarterly

Table 2 Member Control Categories

Age	Black male Interview	Black male Diary	Black female Interview	Black female Diary	Non-Black male Interview	Non-Black male Diary	Non-Black female Interview	Non-Black female Diary
14–17	1		13		25		37	
18–21	2	1	14	7	26	13	38	19
22–24	3		15		27		39	
25–29	4	2	16	8	28	14	40	20
30–34	5		17		29		41	
35–39	6	3	18	9	30	15	42	21
40–44	7		19		31		43	
45–49	8	4	20	10	32	16	44	22
50–54	9		21		33		45	
55–59	10	5	22	11	34	17	46	23
60–64	11		23		35		47	
65+	12	6	24	12	36	18	48	24

Table 3. Consumer Unit Composite Subdomains

Category	Mnemonic	Description
Region	NEAST	Northeast region
	NCENTRAL	North Central region
	SOUTH	Southern region
	WEST	Western region
Sampling Frame	CEN70	1970 Census frame Census address list
	SPECPLAC	Special places frame "Windshield" and special enumerations for primarily rural districts, including trailer parks
	ARSEG	Area segments frame Special enumeration for geographical districts with an unacceptably high fraction of addresses from the Census list that are vacant or have been demolished
	NEWCON	New construction frame Augmentation of the Census address list to include new construction permits from local governments
Tenure	OWNER	Owner consumer units
	RENTER	Renter consumer units, including those in student housing
Family Type	ALL_HW	All husband/wife consumer units
	SPT1+<18	Single-parent consumer units
	SINGLE	Single-person consumer units
	OTHER	All other consumer units

population counts in Table 2, and two correspond to the three monthly total population counts. One monthly total population control constraint was dropped, since the quarterly age/race/sex and the monthly total population controls are each exhaustive for persons and since all age/race/sex constraints were used. X_1^0 has about 2,000 rows corresponding to the Diary sample size. The matrix X_2^0 has 59 columns of which 48 correspond to the Interview classifications in Table 2, and 11 correspond to the 12 rotation/month control constraints, where one of the latter was dropped as redundant. X_2^0 has about 4,500 rows corresponding to the Interview sample size. The matrices X_1^c and X_2^c have 11 columns corresponding to the composite classifications of consumer units given in Table 3, where one category was dropped from the SAMPLING FRAME, TENURE, and FAMILY TYPE classification groupings because each grouping is exhaustive for consumer units and because the four REGION classifications were used. The row dimensions of the X_i^c matrices correspond to those of the X_i^0 matrices above. To relate this in the usual nomenclature of weighting adjustment algorithms, the constraints for both samples together constitute an eight-way table to be adjusted to known marginals.

The vector of weights Ω to be adjusted are the design sample weights adjusted for field subsampling and for nonresponse with the weighting cell factors from the production data base. Our application is an example of one of the Luery (1980, 1986) specifications in concert with the Horvitz–Thompson specification of Bethlehem and Keller (1983) and Zieschang (1985, 1986a,b), in which $\Lambda = \text{diag}(\Omega) - I$. The weighting adjustment equation (3.2) was computed under the definition of variables (3.10) It was assumed that all consumer units in the Interview component of the survey had been independently selected, implying the diagonality of Λ_{22}. The CU's were assumed to be independently selected between survey components, so that $\Lambda_{12} = \Lambda_{21}' = 0$. Finally, in adjusting the weights of diary-weeks, while CU'-s were assumed to be independently selected into the Diary sample, the joint probability of obtaining a diary response in both of the designated weeks for a given CU was not assumed zero. Details of the method by which this probability was estimated can be found in Zieschang (1985). As a result, for more than 80% of the Diary sample CU's there were 2 × 2 matrices corresponding to the two weekly diaries down the diagonal of Λ_{11}, whose elements were otherwise set to 0. The form of the matrix Λ for the Horvitz–Thompson variants of GLS when multiple observations are taken from a given sample unit can be found in Zieschang (1985). To accommodate the resulting nondiagonality, for the Horvitz–Thompson variants of GLS Λ_{11} was Cholesky decomposed as $\Lambda_{11} = L_1 D_1 L_1'$, where L_1 is unit lower triangular and D_1 is diagonal, and X_1^l was transformed as $\tilde{X}_1^l = L_1 X_1^l$ for $l = 0, c$.

5.2 Empirical Results

The data for constructing the sample weights Ω, the weighting matrix Λ, and the regressor matrix X originated from the Bureau of Labor Statistics (BLS) Consumer Expenditure Survey data base. Ω was the product of the base weight, the field subsampling adjustment, and the monthly noninterview adjustment. The X matrix of counts of persons or indicators of subdomain membership was generated from the AGE, RACE, and SEX variables and REGION, CUTENURE, FAM_TYPE, and other variables in the data base.

First, Tables 4 and 5 contain some results for the first quarter of the study period, 1980IV, paralleling the numerical example of Table 1 These tables demonstrate the impact of GLS control and composition on the level and the estimated standard deviations of consumer unit totals

Table 4 Comparison of GLS Variants With Principal Person Weighting Consumer Unit Totals for Composite Subdomains Diary Survey, 1980IV

	Control only				Control and Composition		
Subdomain	Principal Person	GLS	GLS Person Weighted	GLS Person Weighted Over Principal Person	GLS	GLS Person Weighted	GLS Person Weighted Over Principal Person
ALL	70.900,521	67,984,658	69,523,080	70,385,857	66,802,786	67,391,580	68,572,737
	(1,230,184)	(911,326)	(823,104)	(848,856)	(642,079)	(515,931)	(492,635)
Region							
EAST	17 198,791	16,909,410	17,257,699	17,191,603	15,669,277	15,765,310	16,164,182
	(937,501)	(691,581)	(767,483)	(839,701)	(604,810)	(621,617)	(686,730)
NCENTRAL	18,661,173	17,724,192	18,269,606	18,612,837	17,723,265	17,913,527	18,347,299
	(1,227,049)	(1,126,181)	(1,130,014)	(1,177,204)	(894,701)	(852,807)	(832,334)
SOUTH	19,667,850	18,796,987	19,141,763	19,423,039	19,951,374	20,170,465	20,080,961
	(1,021,359)	(821,474)	(861,792)	(881,409)	(1,047,737)	(999,607)	(885,460)
WEST	15,372,708	14,554,068	14,854,012	15,158,368	13,458,860	13,542,279	13,980,295
	(983,417)	(986,344)	(1,060,340)	(1,111,782)	(607,231)	(641,614)	(648,320)
Sample Frame							
CEN70	45,495,340	43,505,013	44,357,038	45,172,623	42,369,739	42,780,514	43,923,409
	(1,624,755)	(1,204,618)	(1,382,117)	(1,634,074)	(1,033,575)	(1,170,261)	(1,185,410)
SPECPLAC	2,706,773	2,361,825	2,574,193	2,741,073	1,502,632	1,470,786	1,588,019
	(827,564)	(626,646)	(613,521)	(761,264)	(456,972)	(383,843)	(383,898)
ARSEG	10,926,297	10,570,195	10,732,477	10,724,526	11,621,864	11,689,251	11,496,350
	(1,632,100)	(1,413,001)	(1,473,402)	(1,634,074)	(1,367,826)	(1,380,345)	(1,407,397)
NEWCON	11,772,112	11,547,626	11,859,372	11,747,636	11,308,551	11,451,029	11,564,959
	(555,509)	(511,311)	(580,010)	(697,683)	(422,045)	(435,627)	(488,236)
Housing Tenure							
OWNER	41,541,102	40,802,643	41,041,665	40,934,577	40,773,338	40,878,754	41,059,692
	(1,140,753)	(961,665)	(950,951)	(1,104,621)	(603,109)	(596,758)	(733,718)
RENTER	29,352,019	27,174,840	28,473,585	29,443,368	26,024,091	26,512,826	27,513,044
	(1,529,159)	(1,443,662)	(1,416,666)	(1,555,822)	(833,646)	(731,015)	(899,193)
Family Type							
ALL_HW	41,244,718	40,941,057	40,862,679	40,800,672	39,513,227	39,452,349	39,987,131
	(1,036,173)	(838,109)	(880,937)	(974,427)	(451,559)	(471,590)	(580,884)
SPT1+<18	3,790,091	3,348,142	3,564,845	3,833,091	3,588,161	3,765,149	3,886,208
	(451,992)	(407,992)	(484,563)	(484,449)	(234,700)	(280,096)	(280,841)
SINGLE	20,705,881	18,890,032	20,264,376	20,723,600	17,972,564	18,486,380	19,086,489
	(1,053,588)	(1,100,177)	(1,084,186)	(1,153,901)	(802,634)	(736,828)	(699,336)
OTHER	5,159,831	4,805,428	4,831,180	5,028,494	5,728,835	5,687,703	5,612,909
	(922,709)	(669,632)	(679,313)	(824,131)	(290,030)	(283,069)	(490,908)

NOTE Estimated standard deviations are in parenthesis These results were generated from weights calculated without recoding negatives

for the composite subdomains of Table 3 for the Diary (Table 4) and Interview (Table 5) portions of the Survey. 1980IV was selected because it was an early quarter from the survey, and displayed noticeable differences in the Principal Person Diary and Interview estimates for ALL CU's and various subdomains, particularly SINGLE and RENTER CU's. The difference for ALL CU's was approximately three million, or 4.1% of the Interview estimate. Although all of the sources of this difference have not been identified, at least one explanation is found in the fact that the quarterly Interview sample contained substantially fewer student consumer units than the Diary sample. Since student consumer units are mostly single-person units, the Diary portion therefore estimated more consumer units in a given population than the Interview portion. To the extent that the difference is explained by random factors, both samples are unbiased, and composite estimation of these totals by bringing the totals of the two samples together via GLS weighting reduces the variance and uniformly improves estimates from the Survey. To the extent that the difference is explained by nonrandom factors, the case for composite estimation in reducing mean squared error is less compelling, but it may still be beneficial.

To highlight the effects of composition, the GLS variants were computed with and without the composite constraints. The first column of each table displays the Principal Person estimates (including marriage adjustment) for ALL CU's and the composite subdomains. The next three columns display the three GLS variants illustrated in the example problem of Table 1, computed without the constraints forcing equality in the total estimates for the composite subdomains, but with the population control constraints on the domains given in Table 2. For comparison with the noncomposite results within the Survey, the last three columns are the same in both tables, displaying

Table 5. Comparison of GLS Variants With Principal Person Weighting Consumer Unit Totals for Composite Subdomains Interview Survey, 1980IV

	Control only				Control and composition		
Subdomain	Principal Person	GLS	GLS Person Weighted	GLS Person Weighted Over Principal Person	GLS	GLS Person Weighted	GLS Person Weighted Over Principal Person
ALL	68,063,128	66,262,352	66,471,861	66,964,169	66,802,786	67,391,580	68,572,737
	(950,658)	(773,295)	(658,354)	(710,264)	(642,079)	(515,931)	(492,635)
Region							
EAST	15,374,558	14,904,749	14,894,674	15,176,243	15,669,277	15,765,310	16,164,182
	(748,813)	(725,787)	(728,894)	(746,305)	(604,810)	(621,617)	(686,730)
NCENTRAL	18,350,489	17,749,641	17,196,774	18,067,400	17,723,265	17,913,527	18,347,299
	(1,060,272)	(1,069,852)	(1,002,745)	(1,009,993)	(894,701)	(852,807)	(832,334)
SOUTH	21,152,945	20,742,882	20,881,453	20,728,690	19,951,374	20,170,465	20,080,961
	(1,287,780	(1,400,274)	(1,326,954)	(1,227,666)	(1,047,737)	(999,607)	(885,460)
WEST	13,185,137	12,865,080	12,898,959	12,991,836	13,458,860	13,542,279	13,980,295
	(674,731)	(600,089)	(660,097)	(602,303)	(607,231)	(641,614)	(648,320)
Sample Frame							
CEN70	42,800,072	41,510,199	41,705,495	42,269,255	42,369,739	42,780,514	43,923,409
	(1,159,997)	(1,162,306)	(1,279,118)	(1,102,120)	(1,033,575)	(1,170,261)	(1,185,410)
SPECPLAC	1,316,890	1,228,346	1,109,283	1,204,721	1,502,632	1,470,786	1,588,019
	(425,545)	(438,025)	(361,541)	(342,295)	(456,972)	(383,843)	(383,898)
ARSEG	12,498,477	12,360,461	12,351,234	12,187,600	11,621,864	11,689,251	11,496,350
	(1,545,930)	(1,672,791)	(1,642,303)	(1,520,873)	(1,367,826)	(1,380,345)	(1,407,397)
NEWCON	11,447,689	11,163,345	11,305,850	11,302,593	11,308,551	11,451,029	11,564,959
	(577,539)	(470,252)	(501,334)	(491,029)	(422,045)	(435,627)	(488,236)
Housing Tenure							
OWNER	41,500,324	40,668,200	40,699,990	40,742,193	40,773,338	40,878,754	41,059,692
	(793,104)	(635,115)	(614,002)	(696,861)	(603,109)	(596,758)	(733,718)
RENTER	26,562,804	25,594,152	26,562,804	26,221,976	26,024,091	26,512,826	27,513,044
	(888,643)	(834,610)	(888,643)	(764,738)	(833,646)	(731,015)	(899,193)
Family Type							
ALL_HW	39,491,130	38,547,338	38,537,712	38,540,613	39,513,227	39,452,349	39,987,131
	(670,080)	(466,383)	(491,428)	(502,052)	(451,559)	(471,590)	(580,884)
SPT1 + <18	3,850,412	3,687,001	3,793,018	3,810,071	3,588,161	3,765,149	3,886,208
	(286,139)	(226,313)	(251,142)	(231,066)	(234,700)	(280,096)	(280,841)
SINGLE	18,210,732	17,610,303	17,773,333	18,186,527	17,972,564	18,486,380	19,086,489
	(931,568)	(941,393)	(861,166)	(837,886)	(802,634)	(736,828)	(699,336)
OTHER	6,510,854	6,417,709	6,367,798	6,426,958	5,728,835	5,687,703	5,612,909
	(410,397)	(364,791)	(365,286)	(384,211)	(290,030)	(283,069)	(490,908)

NOTE Estimated standard deviations are in parenthesis These results were generated from weights calculated without recoding negatives

the results for composite subdomains with the composite equality constraints enforced

The Consumer Expenditure Survey production system is set up to facilitate computation of variances by the Balanced Repeated Replication (BRR) method. In the production data base, the Principal Person weights are computed separately for the full sample and for each of the 20 replicate half-samples into which the quarterly samples are divided for variance estimation purposes. Correspondingly, in this and subsequent computations of variances, the GLS variants were also computed separately for each replicate. The standard deviations were computed as the root mean squared deviation around the mean of the estimates from the replicate samples. BRR variance estimators are unbiased for linear estimators Exact theoretical results for nonlinear estimators such as ratio (Principal Person) and regression (GLS) estimators are few, and the estimates in this study are not corrected for the nonlinearity of either the Principal Person or GLS weighting adjustment procedures. However, Wolter (1985, p. 121) cited a number of evaluations of the performance of BRR for ratio and regression estimators suggesting that the method provides satisfactory variance estimates in these cases. In any case, the BRR results across variants of GLS should be indicative of the relative precision of the total estimators computed under each GLS weighting variant, conditional on the sample All of the assessments of relative precision below do not take into account the sampling variability of the variance estimates themselves, and should therefore be regarded as suggestive but not conclusive

In general, the results from the survey data are similar to the results from the artificial example. The GLS total estimates were all lower than those computed with Principal Person weights. However, their estimated standard deviations were often more than correspondingly lower overall, with the best results in the ALL category obtained for the Person Weighted GLS Over Principal Person Control/Composition variant. This variant was introduced in column 8 of Table 1. Tables 4 and 5 also give some indication of the effect of the "person oriented," consumer unit size-adjusted variant of GLS, as computed for the GLS Person Weighted results of columns 7 and 8 of Table 1. The standard GLS variant generated the closest, but also the lowest, control-only ALL CU totals between the Diary and Interview samples, while the GLS Person Weighted Over Principal Person variant produced a gap comparable to that for Principal Person alone. The GLS Person Weighted total estimates tended to lie between the standard GLS and GLS Over Principal Person totals. For both the Diary and the Interview, the GLS Person Weighted results displayed the best variance performance on the ALL CU's domain when only the control constraints were imposed. The comparison between the methods, particularly standard and Person Weighted GLS, was not as decisive for individual subdomains, however. A comparison of the Principal Person and GLS Person Weighted Over Principal Person columns shows a large reduction in variance for ALL CU's, but generally variable differences in the standard deviation estimates for the subdomain totals.

All of the composite variants display generally notable variance reductions over their noncomposite counterparts. The standard deviations for the composite variants favor either the GLS Person Weighted or GLS Person Weighted Over Principal Person methods. However, the total estimates for these two variants are more than two standard deviations apart. A BLS internally generated estimate of total consumer units used Families and Unrelated Individuals from the March 1980 Current Population Survey as a proxy for total consumer units in the CE population. At approximately 69,255,000, this evidence favors the higher, GLS Over Principal Person estimate.

Zieschang (1985) compared the results from using GLS control/composition with Principal Person methods of weighting adjustment. The ratios of estimates and their BRR coefficients of variation were computed for totals on a variety of consumer unit subdomains for the 13 quarters from 1980IV to 1983IV. To provide a baseline look at the performance of the GLS algorithm, the basic, non-Person Weighted variant of GLS was used. In addition to consumer unit counts, the variable Mean Family Income Before Tax (FIBT) was also compared across the same subdomains and periods to provide some information on the impact of the difference in techniques on a variable closely associated with family expenditures, the focus of the Survey. The first group of subdomains was the column of composite categories in Table 3. A second group was formed from a selection of categories from the "stub" used for BLS Consumer Expenditure Survey publications, as listed in Table 6. Because of the large number of subdomains and time periods, these statistics are presented in Tables 7 and 8 for all subdomains as geometric means of ratios of GLS values to Principal Person values over the quarters of the period covered by the study

Table 7 contains the results for the Diary survey, composite subdomains, consumer unit counts. The GLS results for the ALL category indicate an average 4% reduction from the Principal Person estimates in total consumer units. All subdomains are changed by less than 10%, except the Area Segments (ARSEG) and Special Places (SPECPLAC) frame categories, which show, respectively, about a 34% decrease and a 20% increase. The large changes for these subdomains are the effects of the composition feature of the weighting algorithm in the face of large differences between the surveys on these small frame categories, the Interview being very low in Special Places and the Diary low in Area Segments. In every category other than Special Places, the coefficient of variation is

Table 6. Selected Consumer Unit Publication Subdomains

Category	Mnemonic	Description
Age of the Consumer Unit Head	AGE<25	
	25–34	
	35–44	
	45–54	
	55–64	
	AGE>=65	
Consumer Unit Size	TWO_PER	
	THRE_PER	
	FOUR_PER	
	FIVE_PER	
	SIX+	
Family Type	HW_ONLY	Husband and wife only
	HW OLD<6	Husband/wife with oldest child under 6
	HW·6–17	Husband/wife with oldest child 6–17
	HW 18+	Husband/wife with oldest child over 17
	HW OTHER	Other husband/wife consumer units
Earner Status	SING.0ER	Single, unemployed
	SING.1ER	Single, employed
	CU>2·0ER	Two or more persons, no earners
	CU>2·1ER	Two or more persons, one earner
	CU>2 2ER	Two or more persons; two earners
	CU>2 3+	Two or more persons; three or more earners

Table 7 Consumer Expenditure Survey GLS Versus Principal Person Geometric Means of Quarterly Estimates 1980IV–1983IV. Composite Subdomains

	Ratios of estimates				Ratios of coefficients of variation			
	Total CU's		Mean Family Income Before Tax		Total CU's		Mean Family Income Before Tax	
Subdomain	Diary	Interview	Diary	Interview	Diary	Interview	Diary	Interview
ALL	959	984	1 018	1 006	428	710	922	882
Region								
NEAST	924	1 013	1 029	1 007	549	903	945	974
NCENTRAL	969	974	1 023	1 005	568	860	926	.970
SOUTH	1 023	.957	1 011	1 008	884	879	967	.967
WEST	902	1 007	1 022	1 007	621	805	1 050	.962
Sample Frame								
CEN70	941	998	1 017	1 009	700	769	943	927
SPECPLAC	651	1 196	1 021	978	803	948	1 089	980
ARSEG	1 197	884	1 005	1 014	709	1 105	973	1 051
NEWCON	925	1 014	1 013	1 006	682	935	967	993
Housing Tenure								
OWNER	984	988	1 005	1 006	486	828	990	880
Family Type								
ALL_HW	995	993	1 001	1 006	382	771	974	899
SPT1 + <18	872	952	1 020	1 009	481	796	990	939
SINGLE	898	972	003	1 987	597	988	965	959
OTHER	1.011	982	1 005	1 006	524	802	1 032	1 014

lower, in most cases substantially lower, indicating the beneficial effect on the Diary estimates of composition with the larger Interview Survey. In particular, the evident variance reduction for the ALL category swamps the percent change in the estimate of total consumer units in the population. Table 8 shows the impact of GLS on a subset of stub subdomains that were not selected for composition. Again, there are reductions in coefficients of variation (CV's) for every category. Large variance reductions are indicated for the Age of Head subdomains, as would be expected given the adjustment of the weights to maintain population control totals.

Table 7 also contains results for the Interview Survey, composite subdomains, consumer unit counts. Again, we see notable improvements in precision for all but one of the subdomains, with a nearly 30% reduction in CV's in the ALL category. The Interview results display the reverse of the results for the Diary in the Special Places and Area Segments frames owing to the composition feature of the weighting algorithm. The results for the stub subdomains in Table 8 show a similar pattern of improvement in precision to the results for the Diary, with the control to age population totals notably improving the totals in the Age of Head subdomains.

Finally, Tables 7 and 8 contain GLS/Principal Person ratios of estimates of Mean FIBT for the Diary and Interview surveys on the same composite and stub subdomains. Here, the changes in level and CV's are generally smaller, though noticeable improvements in precision result for the ALL and OWNER categories in the Interview survey, and the Interview estimates are the most beneficially affected by GLS, with generally negligible changes in the estimates of Mean FIBT but reductions in CV's of about 5% or more.

Experience with weights adjusted using either of the variants of the GLS procedure indicated little propensity to generate extremely large weights or proportional weight adjustments relative to those produced by the Principal Person procedure However, occasional downward adjustments did occur that resulted in negative adjusted weights. In this study the "expedient" method was used, with the tolerance interval set with only a lower bound at one fourth of the unadjusted weight. Upper bounds were not enforced because the unadjusted sample weights become progressively downward biased as population growth occurs between sample selections. Secularly rising proportional adjustments are therefore reasonable and upper limits on tolerance regions of weights are potentially risky. Setting only a lower bound will bias upward estimates of totals produced with the adjusted weights; however, evidence from Zieschang (1985) demonstrates that this bias is extremely small for the 25% lower bound used here. Another consideration involves the appropriate tolerance interval lower bound for proportional adjustment of the replicate half-samples that are used for BRR variance computation. The replicate proportional tolerance should be looser than that of the full sample, since the replicate samples are only half the size of the full sample. However, comparisons reported in Zieschang (1985) of BRR coefficients of variation of CU subdomain size estimates generated with the recoded GLS weights, with CV's generated by the unrecoded GLS weights, indicated very little difference for any of the more than 20 subdomains examined over 13 quarters of CE data.

Table 8 Consumer Expenditure Survey GLS Versus Principal Person Geometric Means of Quarterly Estimates 1980IV–1983IV Non-Composite Subdomains

	Ratios of estimates				*Ratios of coefficients of variation*			
	Total CU's		*Mean Family Income Before Tax*		*Total CU's*		*Mean Family Income Before Tax*	
	Diary	*Interview*	*Diary*	*Interview*	*Diary*	*Interview*	*Diary*	*Interview*
Age of Head								
AGE <25	.866	1 002	1 036	983	840	899	1 073	1 067
25–34	971	970	1 012	1 007	632	526	1 045	847
35–44	960	993	1 005	1 008	549	469	1 008	1 018
45–54	999	997	1 007	1 009	613	510	988	914
55–64	1 000	985	1 003	1 006	690	575	917	829
AGE> = 65	947	971	998	1 001	687	652	1 029	1 010
Family Size								
TWO_PER	951	984	994	1 006	722	819	949	908
THRE_PER	.992	994	1 011	1 010	925	877	954	994
FOUR_PER	1 005	991	1 001	1 010	926	960	1 022	963
FIVE_PER	1.013	989	1 023	999	980	941	986	922
SIX +	1 019	.983	1 017	1 012	892	901	988	918
Family Type								
HW_ONLY	952	984	993	1 008	726	806	972	902
HW OLD<6	981	1 002	1 001	.996	929	942	978	935
HW 6–17	.989	997	994	1 006	.842	848	1 022	921
HW.OLD>18	1 069	993	.996	1 006	.942	907	1 019	1 015
HW OTHER	1 130	1.015	1.010	1 002	873	917	1 016	932
Earner Status								
SING 0ER	923	975	1 012	1 000	767	937	1 020	1 006
SING 1ER	884	971	1.010	985	790	981	1 003	972
CU>2 0ER	941	955	1.014	1 002	893	887	937	912
CU>2 1ER	.965	983	996	1 004	935	900	949	889
CU>2 2ER	989	998	995	1 005	877	854	1 031	974
CU>2.3 +	1 076	1 002	1 006	1 001	882	919	994	982

6. CONCLUDING REMARKS AND SUGGESTIONS FOR FURTHER RESEARCH

Least squares weighting adjustment methods are supported by a large literature on generalized regression estimation of finite population statistics. As with any of the other algorithms designed to adjust the cells of a matrix to its marginal totals, the accuracy of the results obtained with GLS weighting adjustment methods depends upon the validity of the underlying model of the unit response and frame coverage process for the actual sampling mechanism. In GLS and other "objective-constraint" algorithms, models of these processes can be incorporated as additional linear constraints on the adjustment of the weights, and in the form of the covariance matrix Λ of the objective function. These methods thus provide a nice organizing structure for submethodologies designed to deal with particular survey problems, whether they arise from the survey operation itself or from the response characteristics of the covered population

A useful feature of the GLS approach is that multiple surveys can easily be linked through their estimates of totals on comparable subdomains by merely imposing additional linear constraints. This was illustrated in the empirical application to the Consumer Expenditure Survey, wherein consumer unit counts were equated between the Diary and Interview samples Of course, methods involving other adjustment objective functions such as raking ratio estimation (RRE) can, in principle, also be modified to perform composition. Further research on combining information from three or more major federal surveys using composite constraints in weighting algorithms could be very fruitful in light of the results obtained for the Consumer Expenditure Survey. Though not highlighted here, this application also demonstrated another useful feature: the capability of performing longitudinal weighting via specification of the off-diagonal elements of the weighting covariance matrix Λ, as in the handling of Diary consumer units with two diary-weeks These off-diagonals involve the probability of obtaining multiple observations from a sample unit· for the Diary, the probability that a CU returned a diary for both survey weeks instead of 0 or 1. Further research might extend this technique, for example, to longitudinally weighting the up to four observations on each CU in annual batches of Interview data. There are also potential applications in other surveys with longitudinal designs, such as the CPS and the Survey of Income and Program Participation (SIPP).

GLS weights can be computed with finite algorithms and thus are, at least in this sense, computationally robust, though it remains to be seen whether GLS weighting possesses substantial advantages over well-implemented alternatives. In a comparison of the GLS and RRE methods for imposing population controls in weighting the Current Population Survey, Copeland, Peitzmeier, and Hoy (CPH) (1988) indicated that GLS was about three times more expensive in computer costs than raking. The source of the difference was not indicated, other than greater file preparation and storage for GLS. On the other hand, Bankier (1990) reported substantial cost advantages for GLS over RRE. The primary computational expense for GLS as implemented in this study was in compiling the cross-products matrices, which may well be the source of the disparity with CPH, since RRE requires a relatively small number of time-consuming multiply operations to complete each iteration. In the CPH study, the number of iterations to acceptable convergence, at 16, was small enough that the relatively low cost per iteration was decisive. However, because the CPS does not include replicate samples for variance estimation, CPH did not need to run the methods independently on replicate samples as was done here for the Consumer Expenditure Survey. These results all pertain to problems containing only control constraints. Results are also needed on the comparative computational performance of alternatives to GLS, particularly RRE, when both control and composite constraints are imposed. Finally, another area for research on computational methods is the bounding of the adjustments to the weights, as discussed by Zieschang (1985), Lemaître and Dufour (1988), and Bankier (1990), and in Section 3.5 on pages 991–992.

GLS is easily implemented with standard fourth-generation software packages. The SUMMARY, CORR, and MATRIX procedures of the Statistical Analysis System (SAS) (SAS Institute 1985) were used in the empirical study above to weight not only the full sample but also 20 replicate samples in the same computer run. (CPH also used SAS MATRIX in their GLS/RRE study for both algorithms.) A less flexible system could have been written using the REG procedure of SAS or any other regression package.

The results from this otherwise fairly standard application of GLS weighting were encouraging, providing similar total and mean estimates to the Principal Person methodology on average, but with pervasive, and in some cases substantial, improvements in the precision of the estimates. It should be added that the variance-improving performance of the GLS technique is comparable with that of other algorithms, based on evidence from artificial data in Alexander (1988) and from the CPS as reported in CPH. At the current state of the art GLS has the advantage over other algorithms of straightforward, flexible, and easily maintainable implementation using existing software components, while generating comparable and demonstrable improvements in precision over currently implemented (non-RRE) ratio estimation methods.

APPENDIX: DERIVATION OF THE FORM OF THE COMPOSITE GENERALIZED REGRESSION ESTIMATOR IN SECTION 3.7

Let

$$\hat{W} = \Omega + \Lambda X(X'\Lambda X)^{-1}(N_X - X'\Omega), \qquad \text{(A.1)}$$

where

$$\Omega = \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}, \qquad \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix},$$

$$X = \begin{bmatrix} X_1^c \\ -X_2^c \end{bmatrix}, \qquad N_X = 0. \qquad \text{(A.2)}$$

The difference between generalized regression estimators for the counts of units on characteristics Y, $\Delta^Y = \tilde{N}_1^Y - \tilde{N}_2^Y = [Y_1' - Y_2']\hat{W}$, can then be written as

$$\Delta^Y = [\hat{N}_1^Y - \hat{N}_2^Y] + [Y_1' - Y_2'] \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ -X_2^c \end{bmatrix}$$

$$\times \left([X_1^{c\prime} - X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ -X_2^c \end{bmatrix} \right)^{-1} (\hat{N}_2^c - \hat{N}_1^c)$$

$$= [\hat{N}_1^Y - \hat{N}_2^Y] + [Y_1' \quad Y_2'] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ -\Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix}$$

$$\times \left([X_1^{c\prime} \quad X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ -\Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \right)^{1} (\hat{N}_2^c - \hat{N}_1^c).$$

If $\hat{\beta}^Y$ is the GLS regression estimator

$$\hat{\beta}^Y = \left([X_1^{c\prime} \quad X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ -\Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \right)^{-1}$$

$$\times [X_1^{c\prime} \quad X_2^{c\prime}] \begin{bmatrix} \Lambda_{11} & -\Lambda_{12} \\ -\Lambda_{12}' & \Lambda_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix},$$

then it follows that the generalized regression estimator for Δ^Y can be written as

$$\tilde{\Delta}^Y = [\hat{N}_1^Y - \hat{N}_2^Y] + \hat{\beta}^{Y\prime}(\hat{N}_2^c - \hat{N}_1^c).$$

The estimator for β^Y corresponds to the model

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1^c \\ X_2^c \end{bmatrix} \beta^Y + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix},$$

where

$$E_\xi \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \operatorname{cov}_\xi \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}' & \Sigma_{22} \end{bmatrix} = \Sigma.$$

If neither of the surveys is longitudinal, so that the probability of selection for each observation is independent, then

$$\Lambda_{ii} = (\operatorname{diag}(W_i))^{1/2}[\Sigma^{-1}]_{ii}(\operatorname{diag}(W_i))^{1/2},$$

$$\Lambda_{ij} = (-\operatorname{diag}(W_i))^{1/2}[\Sigma^{-1}]_{ij}(\operatorname{diag}(W_j))^{1/2}, \qquad \text{where } i \neq j.$$

This reduces to $\Lambda = \operatorname{diag}(W)\Sigma^{-1}$ if the model errors ε are uncorrelated so that Σ is also diagonal. If the model errors are also homoscedastic, then $\Sigma = \sigma^2 I_{(n_1+n_2)}$ and $\Lambda \propto \operatorname{diag}(W)$, the Horvitz–Thompson specification.

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